Chapter 7

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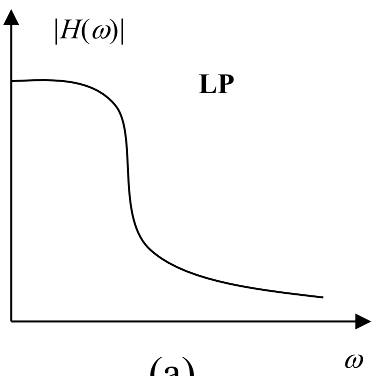
7.1 Introduction

A very important approach to the design of digital filters is to apply a transformation to an existing analogue filter.

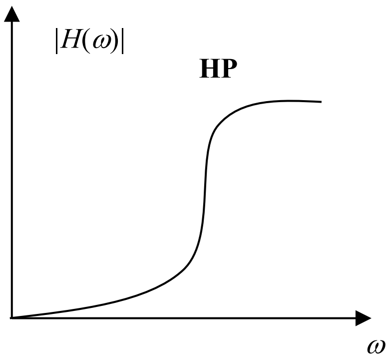
The commonly required types of frequency filter fall into four main groups:

- Low-pass (LP)
- High-pass (HP)
- Band-pass (BP)
- Band-stop (BS)

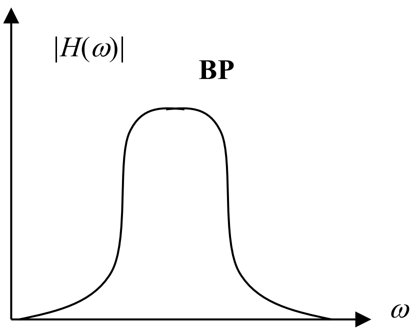
The frequency responses of these filters are shown in Figure **Figure 7.1** (a to d). Also shown are the frequency responses for the **ideal** LP, HP, BP and BS filters.



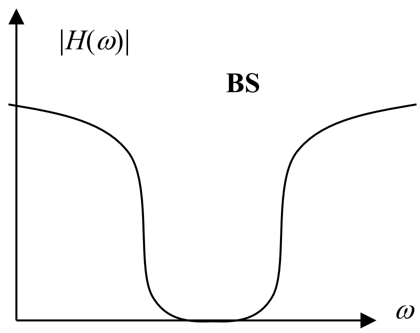
(a)



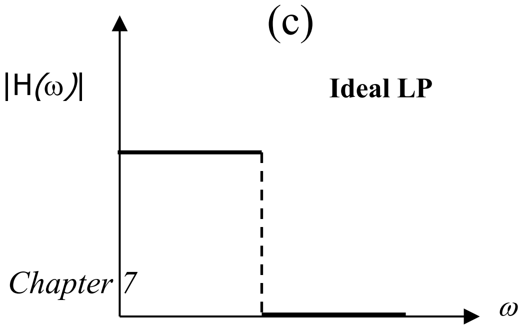
(b)



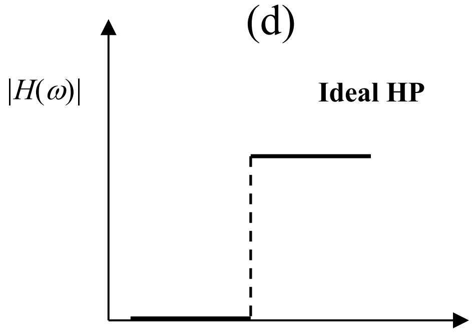
(c)



(d)



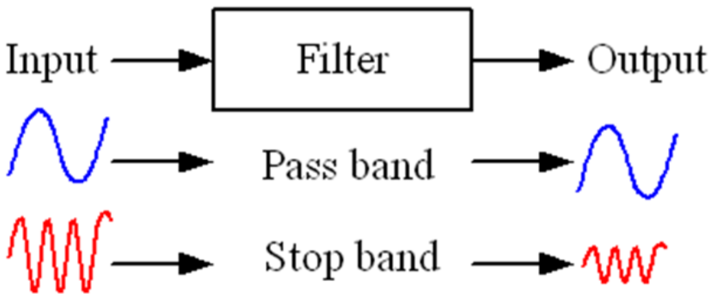
(e)



(f)

The analogue filter design procedure normally begins with a specification of the frequency response for the filter describing how the filter reacts to sinusoid inputs.

If an input sinusoid is not attenuated or attenuated less than a specific tolerance as it goes through the system, it is said to be in a **pass band** of the filter.



If it is attenuated more than a specified value it is said to be stopped and within the **stop band** of the filter.

Input sinusoids with neither a little nor a large amount of attenuation are said to be in the **transition band**. A typical frequency response is shown in **Figure 7.2**, showing the pass band, transition band and stop band.

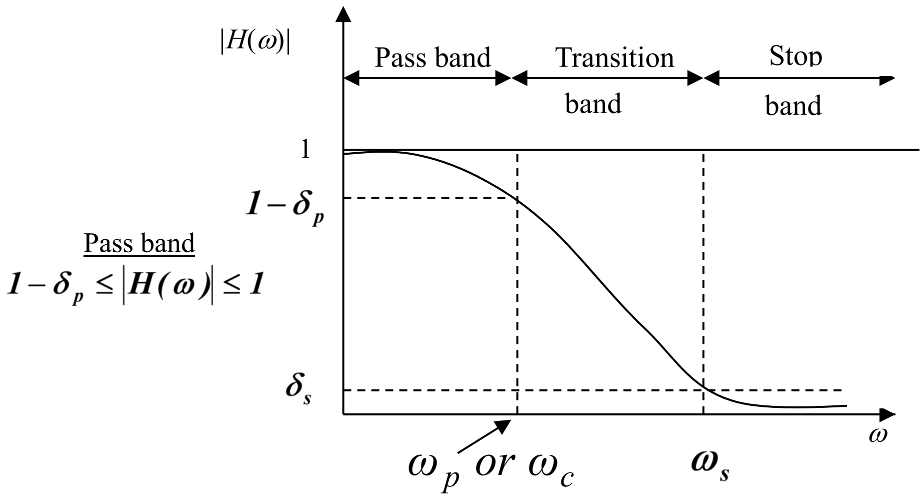


***Figure** Error! No text of specified style in document..**2**: Low-pass filter frequency response*

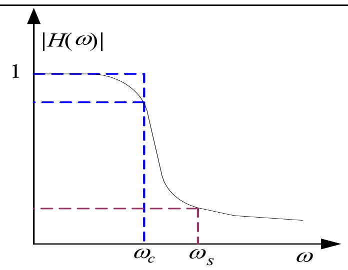
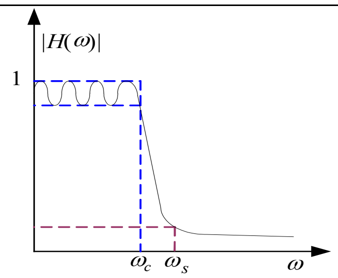
The filter with this type of frequency response is called a low-pass filter as it passes all frequencies less than a certain value ω_p (or ω_c), called the cut-off frequency and attenuates or stops all frequencies past ω_s, the stop band frequency.

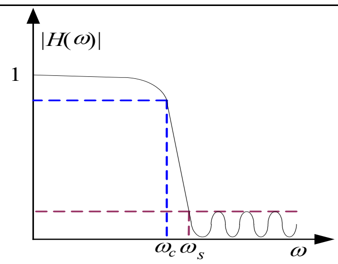
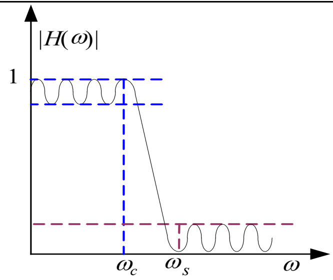
The analogue low pass filters are commonly used as **anti-aliasing** filters that are applied to continuous-time signals before A/D conversion and also used as reconstruction filters.

Note: The technique used to design analogue filters is to specify a prototype low-pass filter function which is normalised to provide a Cut-off frequency (ω_C) at 1 *rad/sec* and then apply transformations to achieve the actual desired cut-off frequencies and filter type.

Therefore, prime consideration will be given to low-pass filter design using the following:

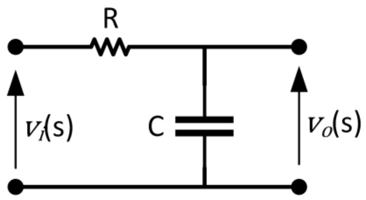
- Butterworth low-pass filter
- Chebyshev I low-pass filter
- Chebyshev II low-pass filter
- Elliptic low-pass filter
- Bessel low-pass filter

Name	Property	
Butterworth	Maximally flat in passband Stopband attenuation relatively poor	
Chebyshev I	Good match with ideal filter characteristic Maximal ripple in passband	

Chebyshev II	Good match with ideal filter characteristic Maximal ripple in stopband	
Elliptic	Minimal transition width Ripple in both passband and stopband	
Bessel	Maximally constant group delay $\{\tau_g = -\frac{d\phi(\theta)}{\theta}\}$	$\phi(\theta) = -a\theta$ *or* $\phi(\theta) = b - a\theta$

7.2 Passive RC filters

- Passive 1st order low-pass filter



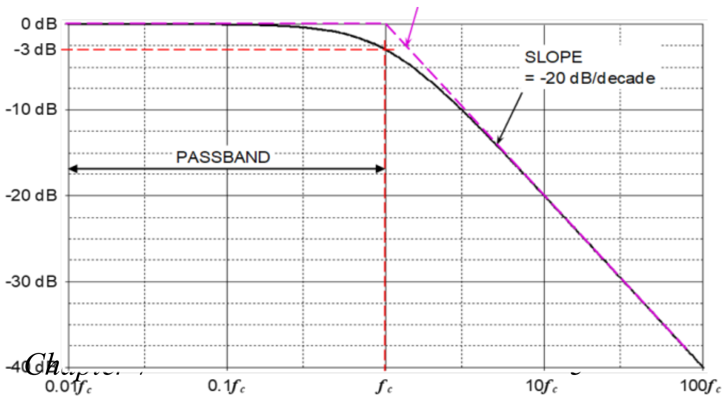
$$H(s) = \frac{v_o(s)}{v_i(s)} = \frac{\omega_c}{s + \omega_c} \quad \text{where } \omega_c = \frac{1}{RC}$$

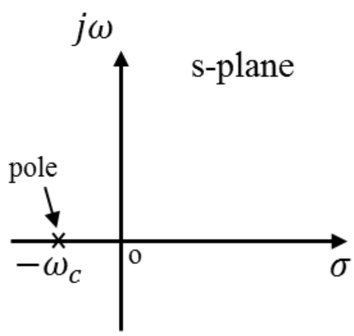
$$H(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_c}}$$

$$|H(j\omega)|_{dB} = 20\, log\left[\frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}\right]$$

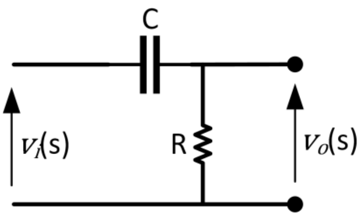
Note

- s - Complex frequency
- $s = j\omega$ sinusoid
- $s = j\omega + \sigma$ Exponential sinusoid
- $s = 0$ dc

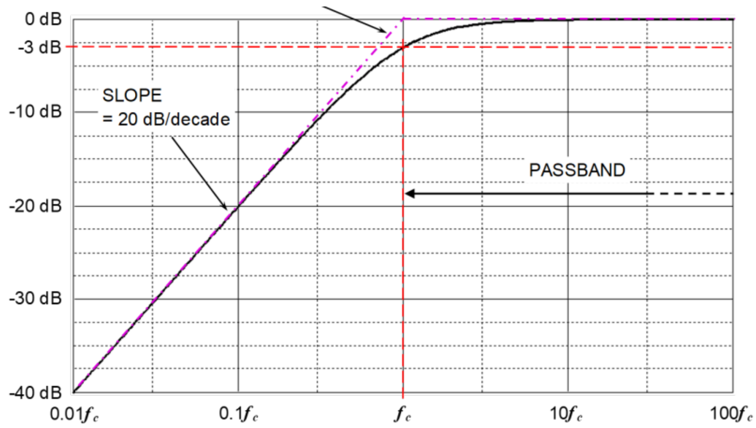


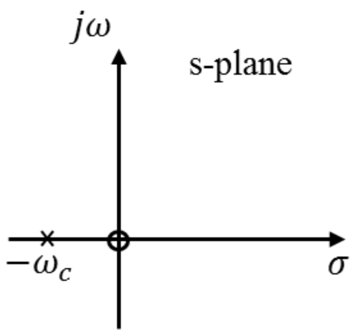


- Passive 1st order high-pass filter



$$H(s) = \frac{v_o(s)}{v_i(s)} = \frac{s}{s+\omega_c} \quad \text{where } \omega_c = \frac{1}{RC}$$





Note

<u>1st order LPF</u>

$$H(s) = \frac{\omega_c}{s+\omega_c}$$

<u>1st order HPF</u>

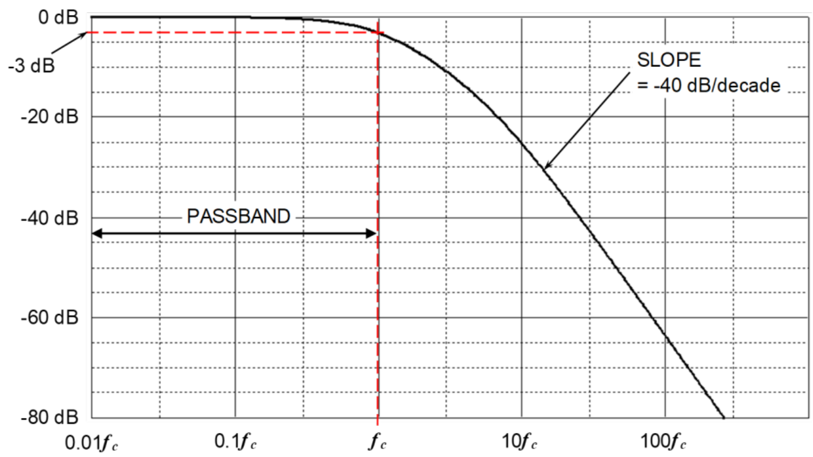
$$H(s) = \frac{s}{s+\omega_c}$$

- Passive 2nd order low-pass filter

$$H(s) = \frac{{\omega_o}^2}{s^2 + b_o s + {\omega_o}^2} \qquad b_o = \frac{\omega_o}{Q}$$

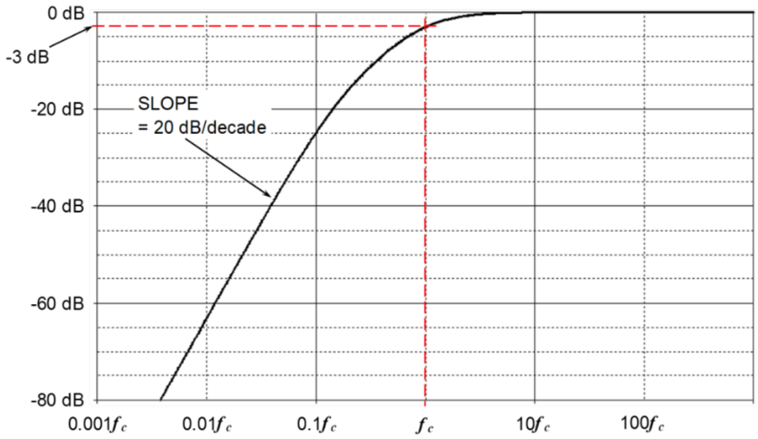
ω_o ← Undamped resonant frequency

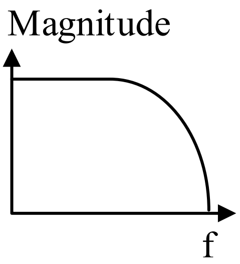
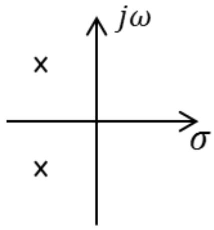
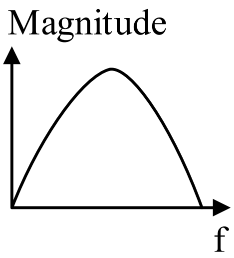
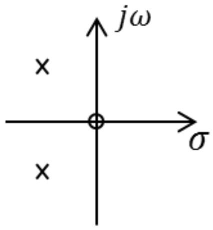
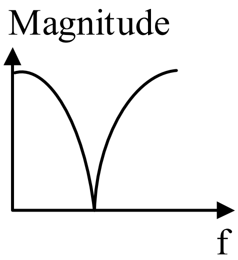
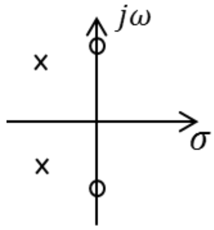
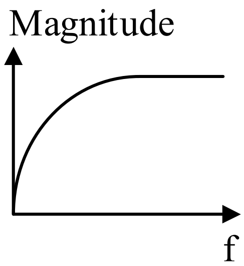
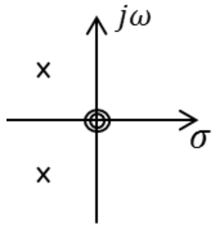
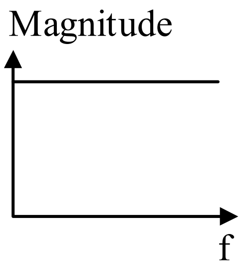
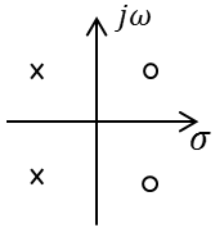
Q ← Q - factor
Q - Quality



- Passive 2nd order high-pass filter

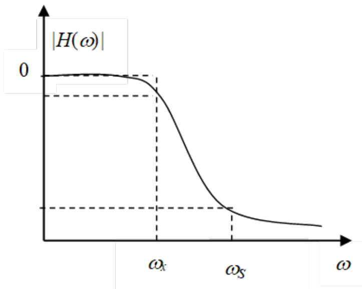
$$H(s) = \frac{s^2}{s^2 + b_o s + \omega_o{}^2}$$



Filter Type	Magnitude	Pole Location	Transfer function
Low-pass			$\frac{{\omega_o}^2}{s^2 + \frac{\omega_o}{Q}s + {\omega_o}^2}$
Band-pass			$\frac{\frac{\omega_o}{Q}s}{s^2 + \frac{\omega_o}{Q}s + {\omega_o}^2}$
Notch (Band-reject)			$\frac{s^2 + {\omega_z}^2}{s^2 + \frac{\omega_o}{Q}s + {\omega_o}^2}$
High-pass			$\frac{s^2}{s^2 + \frac{\omega_o}{Q}s + {\omega_o}^2}$
All-pass			$\frac{s^2 - \frac{\omega_o}{Q}s + {\omega_o}^2}{s^2 + \frac{\omega_o}{Q}s + {\omega_o}^2}$

Note: Lowpass Butterworth Filter

The magnitude response of a Butterworth filter is shown below:



- Lowpass Butterworth filter is a monotonically decreasing function of ω (i.e. throughout pass band and stop band).

The transfer function $H(s)$ is given by:

$$H(s) = \frac{1}{s+1} \qquad 1^{\text{st}} \text{ order}$$

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1} \qquad 2^{\text{nd}} \text{ order}$$

$$H(s) = \frac{1}{(s+1)(s^2 + s + 1)} \qquad 3^{\text{rd}} \text{ order}$$

7.3 Lowpass to High pass transformation

To transform analogue low-pass filter $H(s)$ with unity cut-off frequency to low-pass filter $H(s)$ with cut-off frequency ω_c, we substitute

$$s \to \frac{s}{\omega_c}$$

Example: First order Butterworth prototype filter is given by

$$H(s) = \frac{1}{s+1} \quad \longleftarrow \quad \text{Normalised}$$

To transform to new cut-off frequency $\omega_c = 5$, we replace s with $\frac{s}{\omega_c}$

$$H(s) = \frac{1}{\frac{s}{\omega_c}+1} = \frac{\omega_c}{s+\omega_c} = \frac{5}{s+5}$$

Note: To transform an analogue low-pass filter $H(s)$ with unit cut-off frequency to high-pass filter $H(s)$ with cut-off frequency ω_c, we substitute

$$s \to \frac{\omega_c}{s}$$

Example: First order Butterworth prototype filter is given by

$$H(s) = \frac{1}{s+1} \longleftarrow \text{Normalised}$$

To transfer to a high-pass filter with cut-off frequency $\omega_c = 2$

We replace $s \rightarrow \frac{\omega_c}{s}$; $\quad H(s) = \frac{1}{\frac{\omega_c}{s} + 1} = \frac{s}{s+2}$

Note: *High-pass filter contains zeros as well as poles*

Example:

$$H(s) = \frac{1}{(s+1)(s^2+s+1)} \longleftarrow \text{3rd order Butterworth filter.}$$

Determine the transfer function of the corresponding high-pass filter with cut off frequency $\omega_c = 1$ (normalised)

$$s \rightarrow \frac{\omega_c}{s} \quad \text{i.e.} \quad s \rightarrow \frac{1}{s}$$

normalised

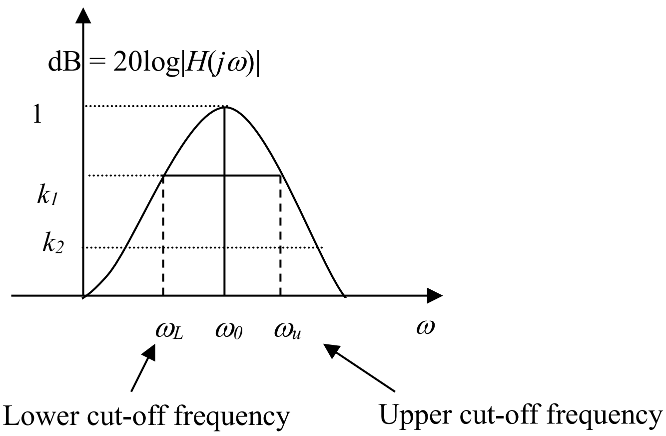
$$H(s) = \frac{1}{\left(\frac{1}{s}+1\right)\left(\left(\frac{1}{s}\right)^2 + \frac{1}{s} + 1\right)} = \frac{s^3}{(s+1)\left(s^2+s+1\right)}$$

7.4 Low-pass to Band-pass Transformations

By definition, a band pass filter rejects both low and high frequency components and passes a certain band of frequencies some where between them. Thus the frequency response $H(j\omega)$, of a band-pass filter has the following properties.

1. $|H(j\omega)| = 0$ at both $\omega = 0$ & $\omega = \infty$

2. $|H(j\omega)| = 1$ for a frequency band centered on ω_0, where ω_0 is the mid frequency of the filter



$$B = \omega_u - \omega_L$$

Bandwidth of the band-pass filter

Method 1: To convert unity cut off low-pass filter $H(s)$ into a Band-Pass filter $H(s)$ with lower cut-off frequency ω_L and the upper cut-off frequency ω_u, we replace

$$s \to \frac{s^2 + \omega_L \omega_u}{s(\omega_u - \omega_L)}$$

Method 2: A lowpass to bandpass transformation can be performed by

$$s \rightarrow \frac{s^2 + \omega_o{}^2}{Bs}$$

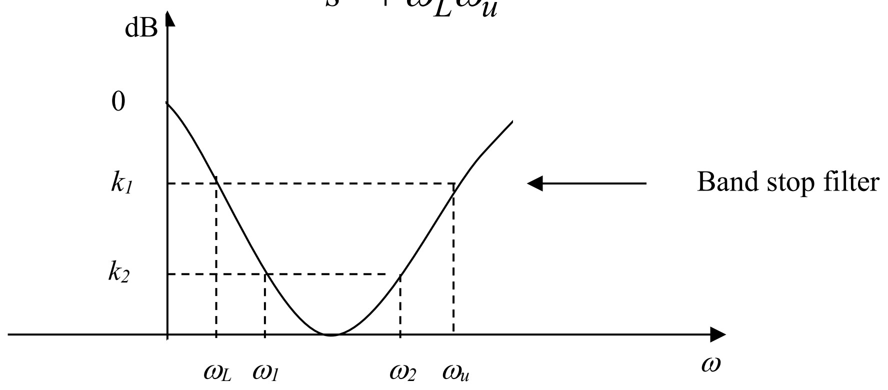
B - Bandwidth of the band-pass filter $(\omega_u - \omega_L)$

ω_0 - centre frequency.

7.5 Low-pass to Band-stop Transformations

Similarly to convert unity cut-off lowpass filter $H(s)$ into a bandstop filter $H(s)$ with lower cutoff frequency ω_L and upper cut-off frequency ω_u, we replace:

$$s \rightarrow \frac{s(\omega_u - \omega_L)}{s^2 + \omega_L \omega_u}$$



Summary: Analogue to Analogue Transformation

Butterworth Prototype response Transformed filter response